Rain or shine? spreadsheet activity

Some parents say it rains more in August when children are not at school than it does in other summer months.

Some people think Scotland is sunnier in spring than England and Wales.

This activity will show you how to use a significance test to decide whether or not such hypotheses are likely to be true.

Information sheet Monthly sunshine

The spreadsheet gives the total monthly hours of sunshine and the total monthly rainfall in England and Wales, Scotland and Northern Ireland for the years 1961–2010.

Think about…

What are the three averages that can be used to represent data?

What measures of spread do you know?

Which are the most appropriate average and measure of spread to use in this context?

A Finding the mean and standard deviation

The sample mean is given by $\bar{x} = \dfrac{\Sigma x}{n}$

sum of the hours of sunshine

number of years

This is the best estimator of the population mean.

To work out the mean on a spreadsheet, use the function **AVERAGE**

The sample standard deviation is given by $s = \sqrt{\dfrac{\Sigma x^2}{n} - \bar{x}^2}$

The best estimator of the standard deviation of the population is

$$\sigma_{n-1} = \sqrt{\frac{n}{n-1}}\, s$$

To find σ_{n-1} on a spreadsheet, use the function **STDEV**

Note

In examinations you will be expected to use your calculator to work out $\bar{x}$ and σ_{n-1}, rather than a spreadsheet.

Try this

Use the data on the EWSun worksheet in the spreadsheet to find, for July then August, the mean and standard deviation of the monthly hours of sunshine. Write your answers in the table below.

Monthly hours of sunshine in England and Wales

	Mean	**Standard deviation**
July		
August		

Think about…

Compare the results.

How can you decide whether July is **significantly** more sunny than August?

Testing the difference between sample means

Distribution of the difference between sample means

When samples of size n_1 and n_2 are taken at random from normal distributions with means μ_1, μ_2 and standard deviations σ_1, σ_2, then the distribution of the difference in sample means, $\overline{X}_1 - \overline{X}_2$,

is normal with mean $\mu_1 - \mu_2$ and standard deviation $\sqrt{\frac{\sigma_1^{\,2}}{n_1} + \frac{\sigma_2^{\,2}}{n_2}}$

The Central Limit Theorem says that this is also true for large samples even if the underlying distributions are not normal.

Summary of method for testing the difference between sample means

1 State the null hypothesis: H_0: $\mu_1 = \mu_2$ (i.e. $\mu_1 - \mu_2 = 0$)
and the alternative hypothesis: H_1: $\mu_1 \neq \mu_2$ for a 2-tailed test
H_1: $\mu_1 < \mu_2$ or H_1: $\mu_1 > \mu_2$ for 1-tailed test

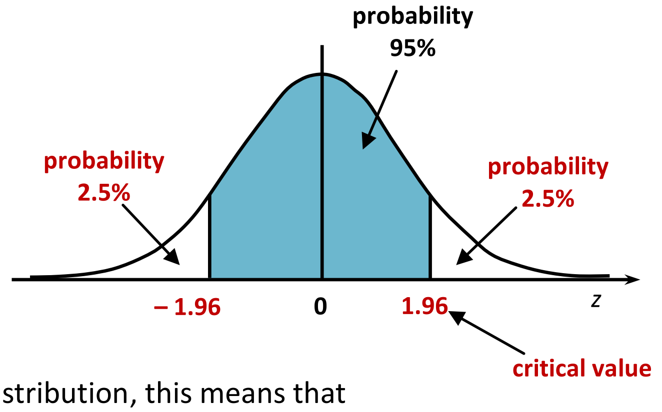
2 Calculate the test statistic:

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

3 Compare the value of the test statistic with the relevant critical value:

2-tailed tests

For a 5% significance test, use $z = \pm$ **1.96**



Think about

Why is 1.96 the critical value?

If the test statistic lies in one of the tails of the distribution, this means that the probability of its occurrence is less than 5% (assuming that the null hypothesis is true). This is an unlikely event, so the null hypothesis should be rejected in favour of the alternative hypothesis.

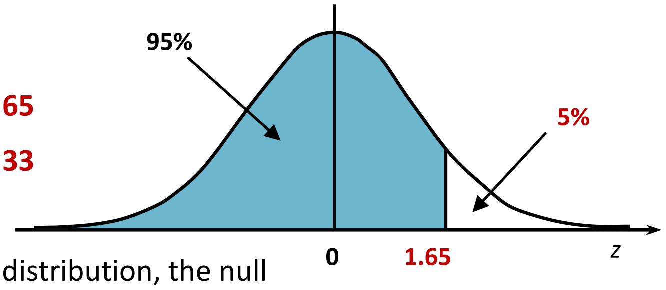
For a 1% significance test, use z = $\pm$ **2.58**

In this case, if the test statistic lies in one of the tails, the event is even more unlikely, and there is even stronger evidence to support the rejection of the null hypothesis.

1-tailed tests

For a 5% significance test, use z = **1.65** or **– 1.65**

For a 1% significance test, use z = **2.33** or **– 2.33**



Again if the test statistic lies in the tail of the distribution, the null hypothesis should be rejected in favour of the alternative hypothesis.

4 State the conclusion clearly.

Testing whether July is significantly more sunny than August

Null hypothesis H$_0$: $\mu_1 = \mu_2$
(i.e. there is no difference in the true population means for July and August)

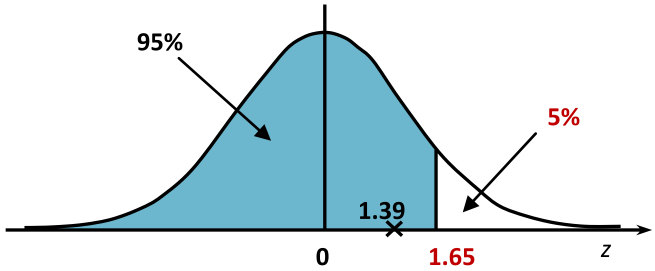
Alternative hypothesis: H$_1$: $\mu_1 > \mu_2$ 1-tailed test
(i.e. on average there is more sunshine in July than August)

Test statistic

$$z = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{184.4 - 174.8}{\sqrt{\frac{37.33^2}{50} + \frac{31.47^2}{50}}}$$

$$z = \frac{9.6}{6.9049...} = 1.39$$

Using a 5% significance test:



1.39 is not in the critical region, so the null hypothesis should be accepted.

Conclusion: July is not significantly sunnier than August.

Try this

Write down and test other hypotheses comparing:

- the amount of sunshine in two other months of the year
- the amount of rainfall in two months
- the amount of sunshine and/or rainfall in the countries of the UK

Write a short report summarising your findings.

Reflect on your work

- What is measured by **standard deviation**?
- When is it better to use σ_n and when σ_{n-1}?
- Describe the steps in a **hypothesis test.**
- What do you do if the value of *z* is in the **critical region?**
- Can you use a hypothesis test to **prove** that a theory is true or false?

Extension

Carry out significance tests on differences in proportions.